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Technical Optimization Techniques for Eliminating Systemic Errors in Data for Digital Construction of Bread Using Image Analysis

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Key words: Image Analysis, Baking Industry, Energy Efficiency

Regarding a scientific knowledge of baking on industrial scale, qualitative information is vast, allowing for high quality products. Quantitative side is, however, left to be desired. Since the latter is what's required for optimization for energy efficiency, this work is a part of an aim to measure and eventually describe physical phenomena before and during baking. The work mentions only one of those variables – measurement of bread dimensions.

Measuring shape and volume changes of a bread before and after baking poses a unique challenge. Measurement has to be accurate and non-destructive as this determines the initial conditions in mathematical modelling. It must be performed in a quick succession, since any delays between proofing and baking changes the end quality of product and drifts away from the real process. Image analysis from pictures taken by camera provides an inexpensive and intuitive way to resolve such intricacies.

Image analysis is performed by taking at least three pictures with a reference of a known length within the picture. Since 2D images inherently provide limited information, the first requirement to achieve desired accuracy is to make sure that the reference is aligned with the plane of view, on which a measured dimension is determined. The second source of error comes from an inability of a camera to see parts of the intersection of measured solid due to perspective or the shape of the obstruction from the solid itself. In given conditions, even with the well-adjusted reference, the relative errors while measuring samples of a length 30 cm can be as high as 10 %.

This work showcases a setup and an apparatus to compensate for the limitations of the camera – it shows a plane of view for a reference and provides an indirect way of outlining the intersection of the sample. The apparatus tested on both laboratory and real-process conditions. The latest iteration showcases relative errors reaching below 1 %. The data can then be used to validate mathematical models of heat transfer in the dough/bread which are then utilized to optimize external conditions to achieve energy savings while maintaining overall quality of the bread.

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This work highlights a setup and an apparatus to compensate for the limitations of the camera – it shows a plane of view for a reference and provides an indirect way of outlining the intersection of the sample. The apparatus evaluated on both laboratory and real-process conditions. The latest iteration displays relative errors reaching below 1 %. The data can then be used to validate mathematical models of heat transfer in the dough/bread which are then utilized to optimize external conditions to achieve energy savings while maintaining overall quality of the bread.

Bread baking is an ancient method of food processing. Throughout the ages, bread spread all over the world and it is now one of the main sources of nutrition of modern civilization[1]. As such, many techniques were developed to improve the quality or the cost of materials. However, given its complexity, few attempts were made to address energy efficiency. To solve that issue, one has to rigorously quantify physical and chemical processes of bread, simulate baking based on initial conditions (initial composition of the bread, its shape, conditions inside the oven…) and optimize said inputs for minimal energy consumption while conserving bread quality. As of this writing, some attempts were made, though these findings were validated on very small samples (250 g at most) [2]. This work aims to provide validating data on volume changes from the real process.

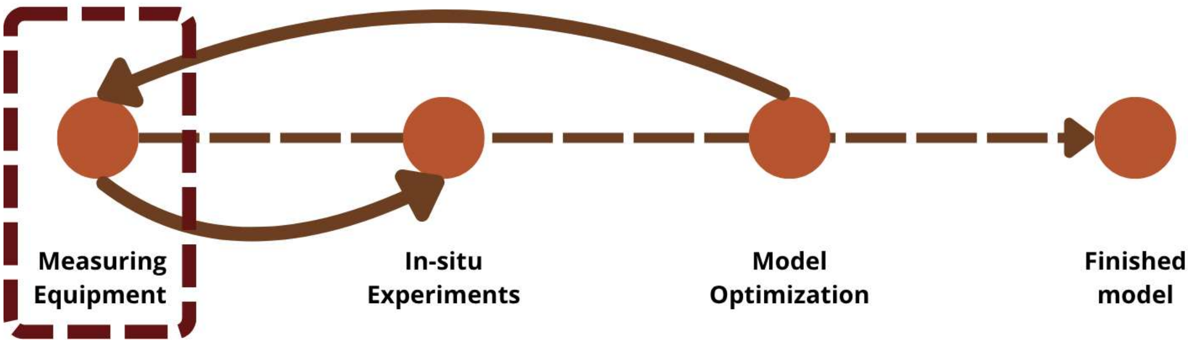


Figure 1: Scheme of the validating process of mathematical models of bread baking.

Measuring volume of a dough/bread poses a unique challenge – since its shape is one of the initial conditions, the method must be non-destructive. Any time between proofing and baking has its influence on its quality, therefore the measurement must be performed in quick succession. Two of the commercially available photogrammetry options were considered. Systems based on near infrared or visible light to determine range require long preparations, for example marking elements, to achieve desired accuracy. Method base on software processing of multiple pictures taken from camera requires even more complex setups – solid colour background, uniform lighting, and that in all angles. Hence, a camera image analysis from a limited number of pictures was used to determine the rough shape and dimensions of an object. This method involves general knowledge of perspective to construct a shape from limited data (at least three pictures from different angles).
Image analysis involves taking a picture of an object with a length reference at sight (see **Figure 2**). With a known real length of the reference, it is possible to correlate distance in the picture to a real one. That said, due to the inherent distortion when projecting a 3D object to 2D, systemic errors are introduced if the measurement is set up without due consideration. After describing its principles, a design of a measurement setup addressing those issues is proposed.

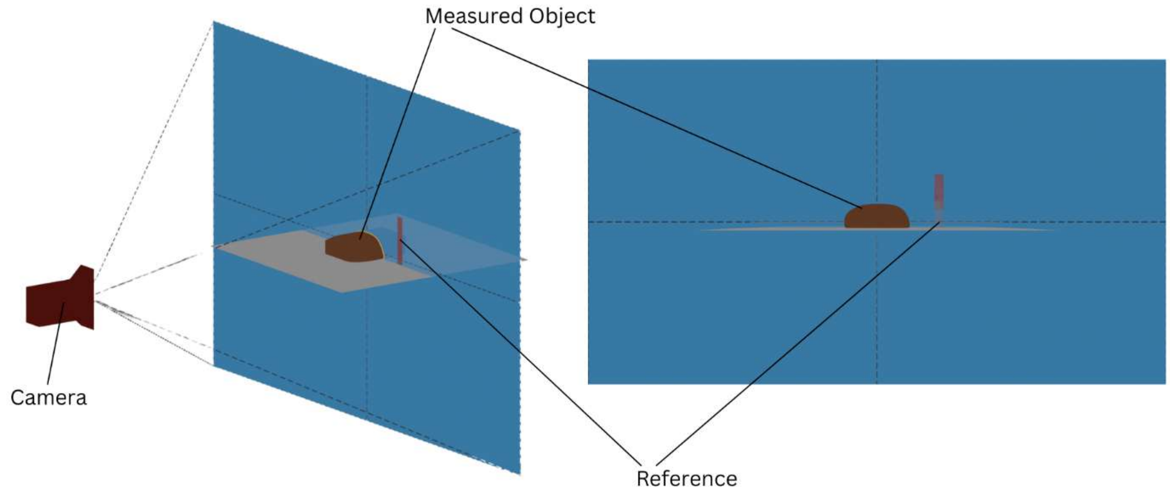


Figure 2: Measurement via taking a picture with a reference and finding correct length by correlating the known length of a reference to the dimensions of the object.

To get a better understanding, a simplified outlook on the problem is presented. A camera sensor can be depicted as a point which senses incoming signal radially.
Determination of the length of an object is done within an image surface (IMS). However, as implied by the camera depiction above, IMS is of a radial nature (and that is if the lens itself does not introduce distortions (is rectilinear) e.g. fisheye effect). A camera signal is in fact a function of view angle (VA) α. In this work, α is an angle on camera from so called principal ray, a line emerging from a camera and set up so that $\alpha = 0°$ is defined as an image centre (see **Figure 3**). But the measured object and reference is straight. Moreover, IMS is hard to accurately determine in reality, given its curved nature in 3D. It is more practical to set physical elements in a scene in a way so that they are within an object plane (OP) which is, implied from its name, planar. OP is perpendicular to the principal ray. If the scene can be reduced to 2D (**Figure 3**, centre), the conversion from the distance of a point in OP from principal ray d_{OP} to VA in an image is:

$$\alpha = \left(\frac{d_{\mathrm{IMS}}}{l_{\mathrm{OP}}}\right) = \tan^{-1}\left(\frac{d_{\mathrm{OP}}}{l_{\mathrm{OP}}}\right), \tag{1}$$

where l_{OP} is the distance of OP from camera and d_{IMS} a length of an arc of a radius l_{OP} (which means IMS and OP share a point, see **Figure 3**). from principal ray. For convenience, from this point, the graph of this function will be named VA distortion graph. Note that the conversion from IMS to OP is called gnomonic projection, used in geography.

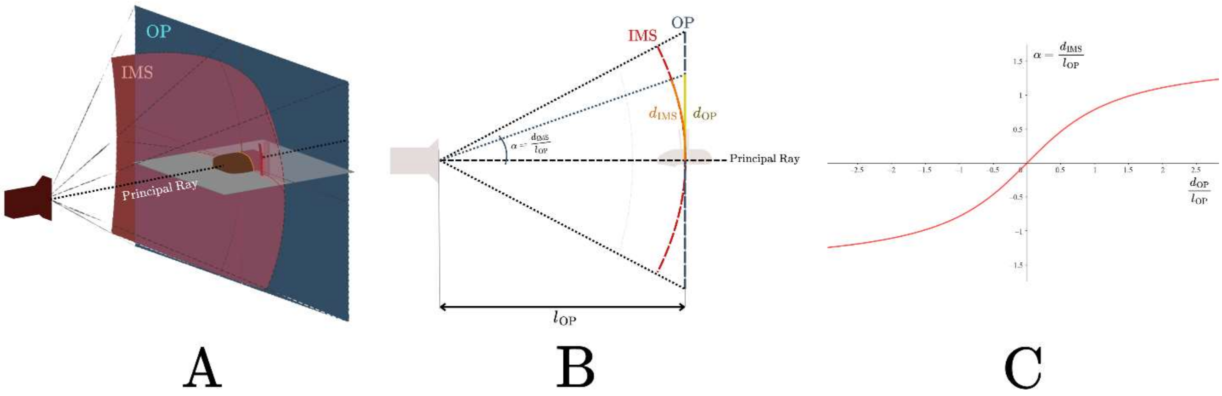


Figure 3: The diagram of the IMS and OP, and its partial representation in a 1D view angle distortion graph (A – 3D scene, B – Side view, C – VA distortion graph of a side view).

To sum up, while moving and working with an object, a reference or a camera in a scene, the elements operate in OP; While digitally determining length, any decision is operating within IMS. The systemic error comes from an assumption that IMS is planar and therefore the same as OP. In the 1D VA distortion graph, the error can be depicted by a difference between function (1) and a function below:

$$\alpha = \left(\frac{d_{\mathrm{IMS}}}{l_{\mathrm{OP}}}\right) \approx \left(\frac{d_{\mathrm{OP}}}{l_{\mathrm{OP}}}\right), \tag{2}$$

which is a representation of the OP (see **Figure 4**) for VA in radians. For example, a standard camera in 1x zoom has a focal length of 23 mm. The maximum horizontal angle of view is about $\alpha_{\mathrm{MAX}} = 38.1° = 0.664$ rad. If taking a picture of an object so that its length horizontally covers 80 % of the width of the image in a landscape mode, therefore an angle of view $\alpha = 0.8\,\alpha_{\mathrm{MAX}} = 30.4° = 0.531$ rad on both sides, the relative distortion of an object by VA in the image is:

$$\delta_{\mathrm{VA}} = \frac{\left(\frac{d_{\mathrm{IMS}}}{l}\right) - \left(\frac{d}{l}\right)}{\left(\frac{d}{l}\right)} = \frac{\left(\frac{d_{\mathrm{IMS}}}{l}\right)}{\left(\frac{d}{l}\right)} - 1 = \frac{\alpha}{\tan(\alpha)} - 1 = \frac{0.531}{\tan(0.531)} - 1 \approx -0.095, \tag{3}$$

which roughly means an object in the IMS will appear -9.5 % smaller than if the IMS was truly planar.

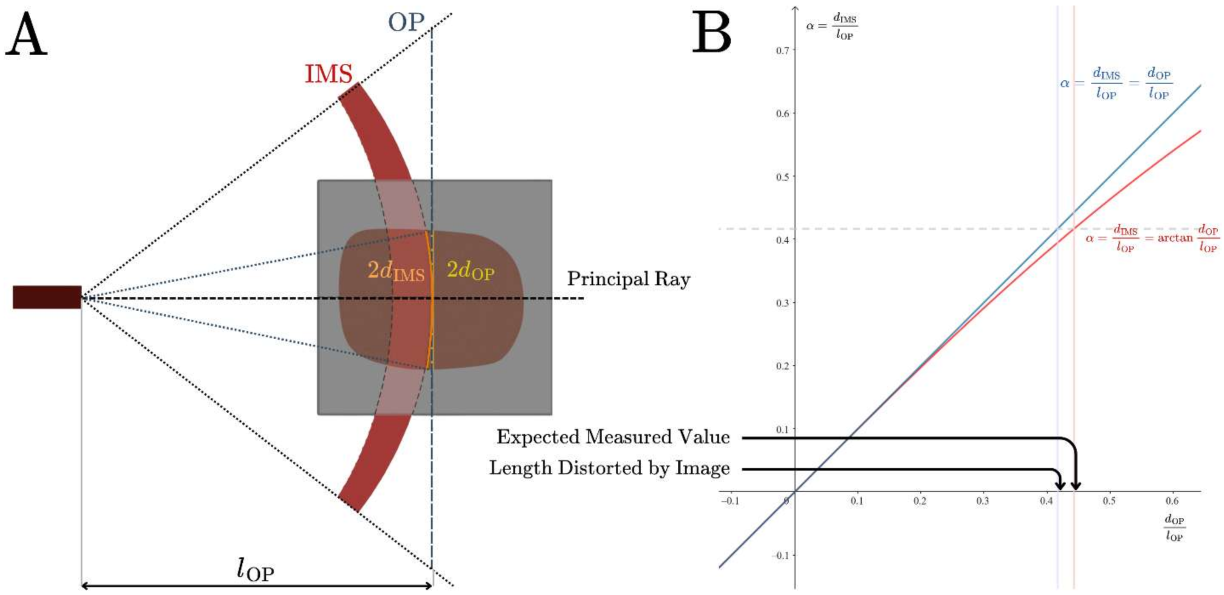


Figure 4: The diagram of the IMS and OP and depicted systemic error from assuming IMS to be planar (same as OP).

From this observation we can deduce we can correlate distance in pixels to a real length without any major errors only if $\frac{d}{l}$ is small (in practice, if the camera is far enough from the measured object). For reference, to achieve relative distortion δ below 1 %, the $\left(\frac{d}{l}\right)$ can be calculated from (3) by substituting $\left(\frac{d_{\mathrm{IMS}}}{l}\right)$ by (1):

$$\delta_{\mathrm{VA}} = \frac{\left(\frac{d_{\mathrm{IMS}}}{l}\right)-\left(\frac{d}{l}\right)}{\left(\frac{d}{l}\right)} = \frac{\tan^{-1}\left(\frac{d}{l}\right)-\left(\frac{d}{l}\right)}{\left(\frac{d}{l}\right)} = \frac{\tan^{-1}\left(\frac{d}{l}\right)}{\left(\frac{d}{l}\right)} - 1, \tag{4}$$

$$-0.01 \geq \frac{\tan^{-1}\left(\frac{d}{l}\right)}{\left(\frac{d}{l}\right)} - 1. \tag{5}$$

By solving for $\left(\frac{d}{l}\right)$, it is found that the ratio must be smaller or equal to approx. 0.175, that is, if an object is 30 cm long, the camera must be at least $\frac{15\ \mathrm{cm}}{0.175} = 85$ cm far from an object, or for an object to cover less than about 32.57 % of the horizontal length of an image in landscape mode. However, it is possible to lower such requirement by a strategically placed reference.

Placing the reference in the OP can be represented in the graph in **Figure 5C**. The referenced points on an image generally create a reference arc of a circle of radius smaller or equal to l in IMS. The object length is then correlated to the length of a reference by comparing their projected arc lengths:

$$\frac{d_{\mathrm{R,OP,2}}-d_{\mathrm{R,OP,1}}}{d_{\mathrm{R,IMS,2}}-d_{\mathrm{R,IMS,1}}} = \frac{d_{\mathrm{R,OP,2}}-d_{\mathrm{R,OP,1}}}{d_{\mathrm{R,IMS,2}}-d_{\mathrm{R,IMS,1}}}, \tag{6}$$

where $d_{\mathrm{R,OP}}$ and $d_{\mathrm{R,IMS}}$ is the distance of a reference point from the principal ray on the reference plane (RP) and IMS respectively, index O denotes the position belongs to the measured object and indexes 1 and 2 two endpoints of chosen lengths.

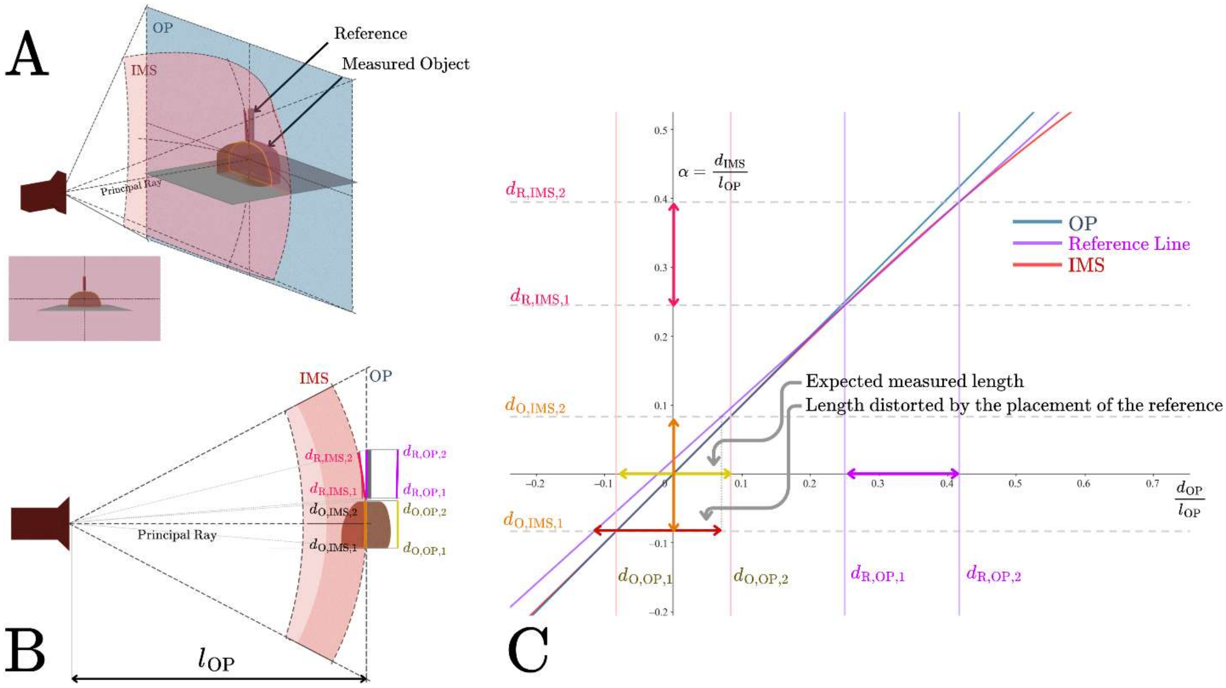


Figure 5:A – Measurement with a Reference of known length. B – side view with highlighted important variables, C –1D representation of the reference in the VA distortion graph and its role in length determination, the gap between the object (yellow line) and reference (purple line) is exaggerated for visibility.

If the reference is not placed in the same OP the object is measured in, reference cannot be used as is. The distortion of the points in the direction from principal ray can be represented by the equation below:

$$\left(\frac{d_{\mathrm{R,RP}}}{l_{\mathrm{RP}}}\right) = \left(\frac{d'_{\mathrm{R,OP}}}{l_{\mathrm{OP}}}\right), \tag{7}$$

where $d_{\mathrm{R,RP}}$ is the distance of a point on RP from the principal ray, l_{RP} the distance of a RP from the camera, $d'_{\mathrm{R,OP}}$ the projected distance of a point to the OP and l_{OP} the distance of the OP from the camera. To put it into perspective, another example of a scene is presented (see **Figure 6**). An object with a vertical length of 10 cm is measured. A reference with a length of 10 cm as well is placed right above the object. The camera is $l = 60$ cm away from the center of the object. If disregarding systemic errors from the angle view distortion described above, the scene set up should be sufficient to measure the object accurately. However, if the reference is placed in a different plane 1 cm closer to the camera than the OP (that is, $l_{\mathrm{RP}} = 59$ cm), given that the coordinates of referenced points are $d_{\mathrm{R,RP,1}} = 5$ cm and $d_{\mathrm{R,RP,2}} = 15$ cm, the resulting projected length $d'_{\mathrm{R,OP,2}} - d'_{\mathrm{R,OP,1}}$ is:

$$\left(\frac{d_{\mathrm{R,RP,1}}}{l_{\mathrm{RP}}}\right) = \left(\frac{d'_{\mathrm{R,OP,1}}}{l_{\mathrm{OP}}}\right) \Rightarrow d'_{\mathrm{R,OP,1}} = d_{\mathrm{R,RP,1}}\frac{l_{\mathrm{OP}}}{l_{\mathrm{RP}}} = 5 \cdot \frac{60}{59} = 5.08 \text{ cm}, \tag{8}$$

$$\left(\frac{d_{\mathrm{R,RP,2}}}{l_{\mathrm{RP}}}\right) = \left(\frac{d'_{\mathrm{R,OP,2}}}{l_{\mathrm{OP}}}\right) \Rightarrow d'_{\mathrm{R,OP,2}} = d_{\mathrm{R,RP,2}}\frac{l_{\mathrm{OP}}}{l_{\mathrm{RP}}} = 15 \cdot \frac{60}{59} = 15.25 \text{ cm}, \tag{9}$$

$$d'_{\mathrm{R,OP,2}} - d'_{\mathrm{R,OP,1}} = 15.25 - 5.08 = 10.17 \text{ cm}. \tag{10}$$

The relative error is:

$$\delta_{RP} = \frac{(d'_{\mathrm{R,OP,2}} - d'_{\mathrm{R,OP,1}}) - (d_{\mathrm{R,RP,2}} - d_{\mathrm{R,RP,1}})}{(d_{\mathrm{R,RP,2}} - d_{\mathrm{R,RP,1}})} = \frac{(d'_{\mathrm{R,OP,2}} - d'_{\mathrm{R,OP,1}})}{(d_{\mathrm{R,RP,2}} - d_{\mathrm{R,RP,1}})} - 1 = \frac{10.17\ \mathrm{cm}}{10\ \mathrm{cm}} - 1 = 0.0169, \quad (11)$$

meaning the reference in the image appears 1.69 % bigger than if placed correctly. With a well-placed reference, the length of the measured object would be the same as the reference, 10 cm (disregarding angle of view distortion, as stated above), meaning $d_{\mathrm{OP}} = 5$ cm from the center. However, by misplacing the reference closer to the camera by 1 cm, the length of the object measured on image $d'_{\mathrm{O,OP}}$ would be smaller:

$$d'_{\mathrm{O,OP}} = d_{\mathrm{O,OP}} \cdot \frac{d_{\mathrm{R,RP,2}} - d_{\mathrm{R.RP,1}}}{d'_{\mathrm{R,OP,2}} - d'_{\mathrm{R,OP,1}}} = d_{\mathrm{OP}} \cdot \frac{1}{\delta_{RP} + 1} = 5 \cdot \frac{1}{0.017 + 1} = 4.91\ \mathrm{cm}, \quad (12)$$

The relative distortion of an object by a misplaced reference plane (MRP) δ_{MRP} is:

$$\delta_{MRP} = \frac{d'_{\mathrm{O,OP}}}{d_{\mathrm{O,OP}}} - 1 == \frac{4.91\ \mathrm{cm}}{5\ \mathrm{cm}} - 1 = -0.0167, \quad (13)$$

that is, the object is measured 1.67 % smaller than normally. Note, that the total error would be higher since in this case, the highest angle view distortion is:

$$\delta_{\mathrm{VA}} = \frac{\tan^{-1}\left(\frac{d_{\mathrm{RP,2}}}{l_{RP}}\right)}{\left(\frac{d_{\mathrm{RP,2}}}{l_{RP}}\right)} - 1 = \frac{\tan^{-1}\left(\frac{15\ \mathrm{cm}}{59\ \mathrm{cm}}\right)}{\left(\frac{15\ \mathrm{cm}}{59\ \mathrm{cm}}\right)} - 1 = -0.0207 = -2.07\ \%. \quad (14)$$

To account for the error, calculations must operate with $d'_{\mathrm{RP,1}}$, $d'_{\mathrm{RP,2}}$ and d_{OP} converted to their respective arc lengths in IMS. In practice, the degree of displacement is often not known and therefore the calculation above serves more as a demonstration. For this reason, the solution will be left for readers as an exercise.

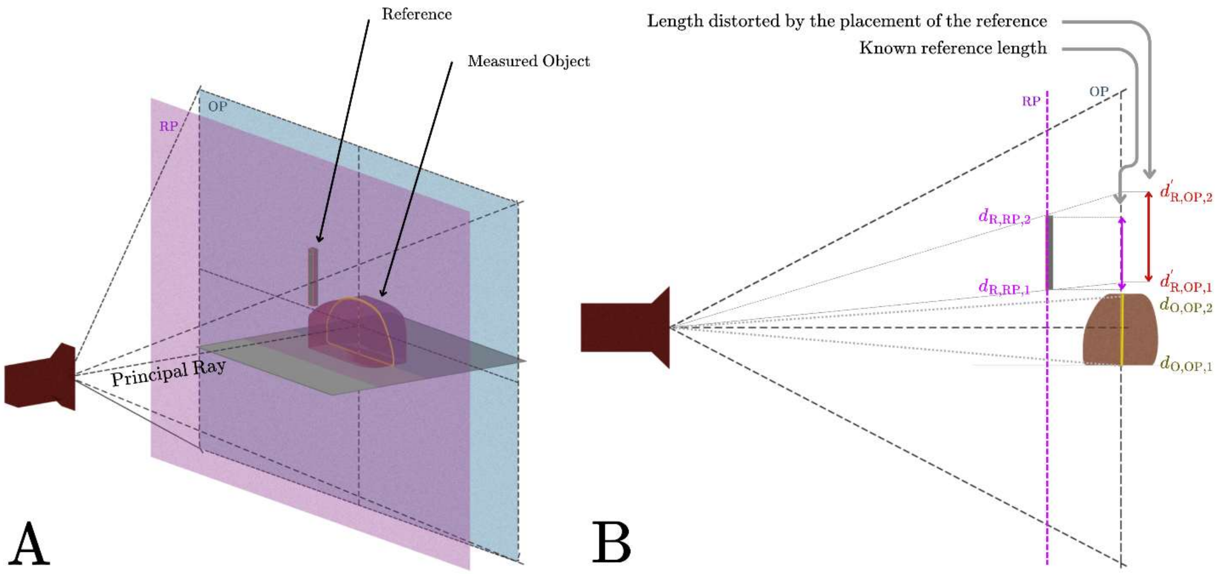
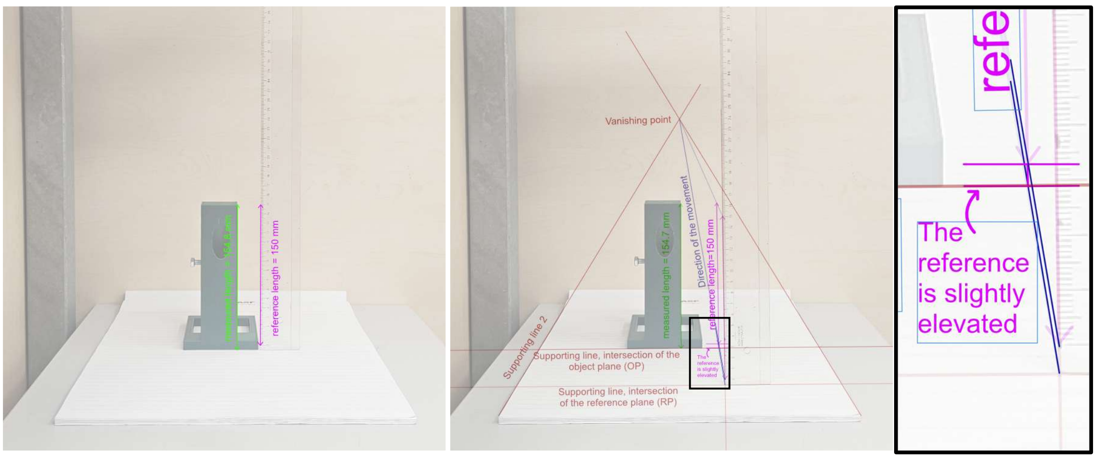


Figure 6: Diagram of the systemic error in length measurement via Image analysis, for the case of reference misplacement.

If a misplaced reference plane error occurs, in most cases, it can't be easily adjusted for in software, if the misplacement of the reference is detected in the first place. However, there are some exceptions which can be corrected digitally (see **Figure 7**). The key to the fix is an understanding how the reference changes in the image if it moves to OP. For this technique to work, both the object measured dimension, and a reference must be parallel to OP (in practice usually satisfied), there are elements in the image from which OPs of both object and reference can be determined; and elements which allow for drawing at least 2 lines in the direction of the movement. Such technique comes from the knowledge of 1-point perspective used predominantly in art, architecture, or graphic design; hence it is easier to understand it by showing a real scenario.

There are two scenes for measured objects **Figure 7A** and **8B**, the latter with a misplaced reference. Note that the first requirement is fulfilled by getting camera levelled against the ground. Another thing to focus on is that the front face of both the object and the reference are intersections of OP and RP, respectively. By comparing bottom horizontal lines derived from the intersections, the misplacement is visible. It works because the elements stand on a shared plane parallel to the ground. On the same plane there is an intersection of a block of paper placed in a way, that the side edges denote a direction of a depth movement. If these lines are elongated, it is shown that all lines of the same direction intersect at the same vanishing point. From this observation a direction of movement (blue lines) for the reference endpoints are constructed. The intercept of the bottom blue line and a bottom line on OP is place a reference must move to land on OP. The resulting length of is the length to be referenced.

From this demonstration it is evident that some mastery of the 1-point perspective is required to effectively wield this technique. It is easier therefore to make sure that the reference lays in the OP.



A B C

Figure 7: A demonstration of the 1-point perspective trick to move the reference to the object plane.

Another thing to consider is the question of the appropriate length of the reference. If the camera distance from the object is great enough (see δ_{VA} section above), this error becomes negligent. However, sometimes working conditions do not allow for enough space. In such cases, if the reference is too big, the distortion can introduce errors from the angle of view distortion. Too small a reference might in addition encounter a camera resolution problem. The best approach is for the reference to be of similar length. To get an idea of the magnitude of such error, a scenario like the example above will be presented.

In **Figure 8**, Two sticks denoting known distance $\Delta d_{\mathrm{R,OP}} = d_{\mathrm{R,OP,2}} - d_{\mathrm{R,OP,1}}$ serve as a reference. A $\Delta d_{\mathrm{O,OP,real}} = d_{\mathrm{O,OP,2}} - d_{\mathrm{O,OP,1}} = 20$ cm wide bread is placed between them in the middle to be measured. The working space does not allow for a camera to be more than $l_{\mathrm{OP}} =$ 60 cm from the center of the bread. If the sticks are just touching the bread, the measured width would be $\Delta d_{\mathrm{O,OP,meas}} = 20$ cm. If $\Delta d_{\mathrm{R,OP}} = 50$ cm, then the measured width in the image will be:

$$\Delta d_{\mathrm{O,IMS}} = d_{\mathrm{O,IMS,2}} - d_{\mathrm{O,IMS,1}} = l_{\mathrm{OP}}\left(\arctan\left(\frac{d_{\mathrm{O,OP,2}}}{l_{\mathrm{OP}}}\right) - \arctan\left(\frac{d_{\mathrm{O,OP,1}}}{l_{\mathrm{OP}}}\right)\right) = 60\left(\arctan\left(\frac{10}{60}\right) - \arctan\left(\frac{-10}{60}\right)\right) = 19.82 \text{ cm} \tag{14}$$

$$\Delta d_{\mathrm{R,IMS}} = d_{\mathrm{R,IMS,2}} - d_{\mathrm{R,IMS,1}} = l_{\mathrm{OP}}\left(\arctan\left(\frac{d_{\mathrm{R,OP,2}}}{l_{\mathrm{OP}}}\right) - \arctan\left(\frac{d_{\mathrm{R,OP,1}}}{l_{\mathrm{OP}}}\right)\right) = 60\left(\arctan\left(\frac{25}{60}\right) - \arctan\left(\frac{-25}{60}\right)\right) = 47.37 \text{ cm}$$

$$\frac{\Delta d_{\mathrm{O,OP,meas}}}{\Delta d_{\mathrm{O,IMS}}} = \frac{\Delta d_{\mathrm{R,OP}}}{\Delta d_{\mathrm{R,IMS}}} \Rightarrow \Delta d_{\mathrm{O,OP,meas}} = \Delta d_{\mathrm{R,OP}} \frac{\Delta d_{\mathrm{O,IMS}}}{\Delta d_{\mathrm{R,IMS}}} = 50 \cdot \frac{19.82}{47.37} = 20.92 \text{ cm} \tag{15}$$

$$\delta_{DRL} = \frac{\Delta d_{\mathrm{O,OP,meas}}}{\Delta d_{\mathrm{O,OP,real}}} - 1 = \frac{20.92}{20} - 1 = 0.0458 \tag{16}$$

The relative distortion by a distortion of a reference length is for this case $\delta_{DRL} = 4.58\,\%$, meaning the distortion caused a measured length to be 4.58 % longer than in reality.

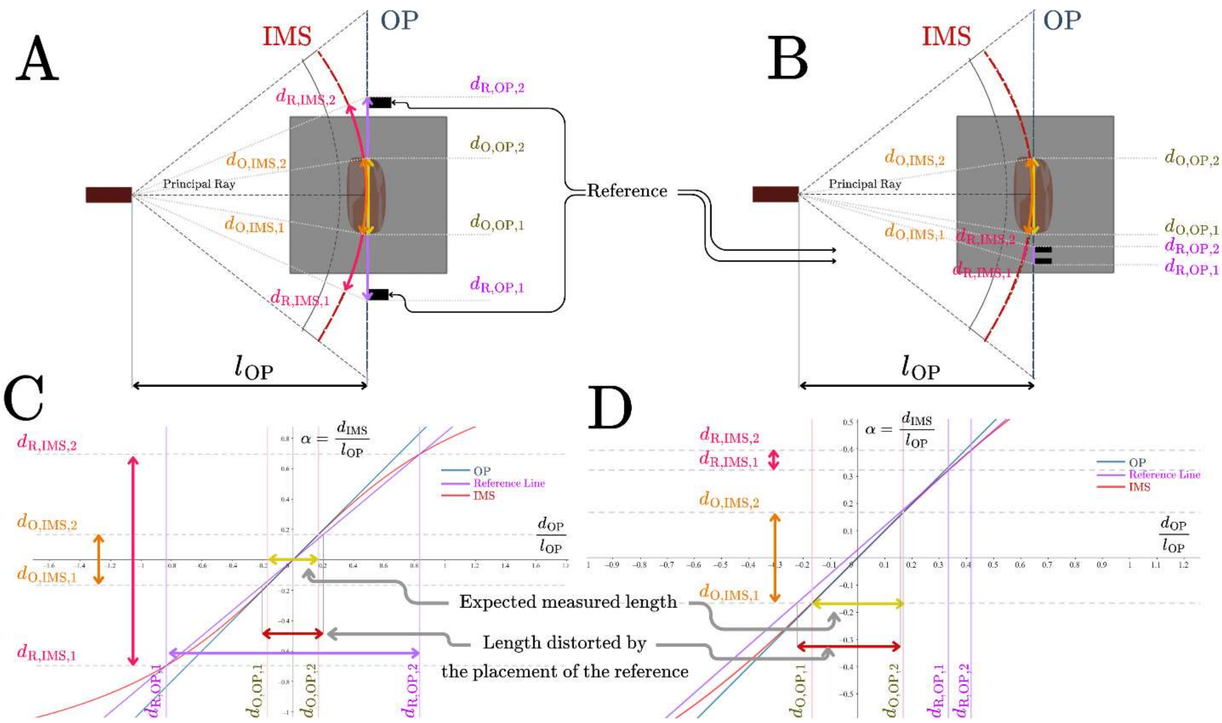


Figure 8: The error from the length and placement reference (A - too big, B – small and too far away from the measured object) and its representation in the VA distortion graph (C and D, exaggerated proportions for visibility).

Important to note that δ_{DRL} is more dependent on the placement of both the reference and the object. The same object is differently distorted in different distances from image centre in IMS. It is important for the reference to not be too far from the measured object (see **Figure 9**). If the reference instead on the side ($d_{\mathrm{R,OP,2}} - d_{\mathrm{R,OP,1}} = (60 - 10)$ cm), with the same equations (14), (15) and (16) the $\delta_{DRL} = 33.13$ %. If the reference were of the same of the same length the error would be 10.65 %. To sum it up, it is important for the reference to be as close to the measured object as possible. To get even better results, it is recommended to place the elements so that they are equally distant from the image centre as their distortions would cancel each other.

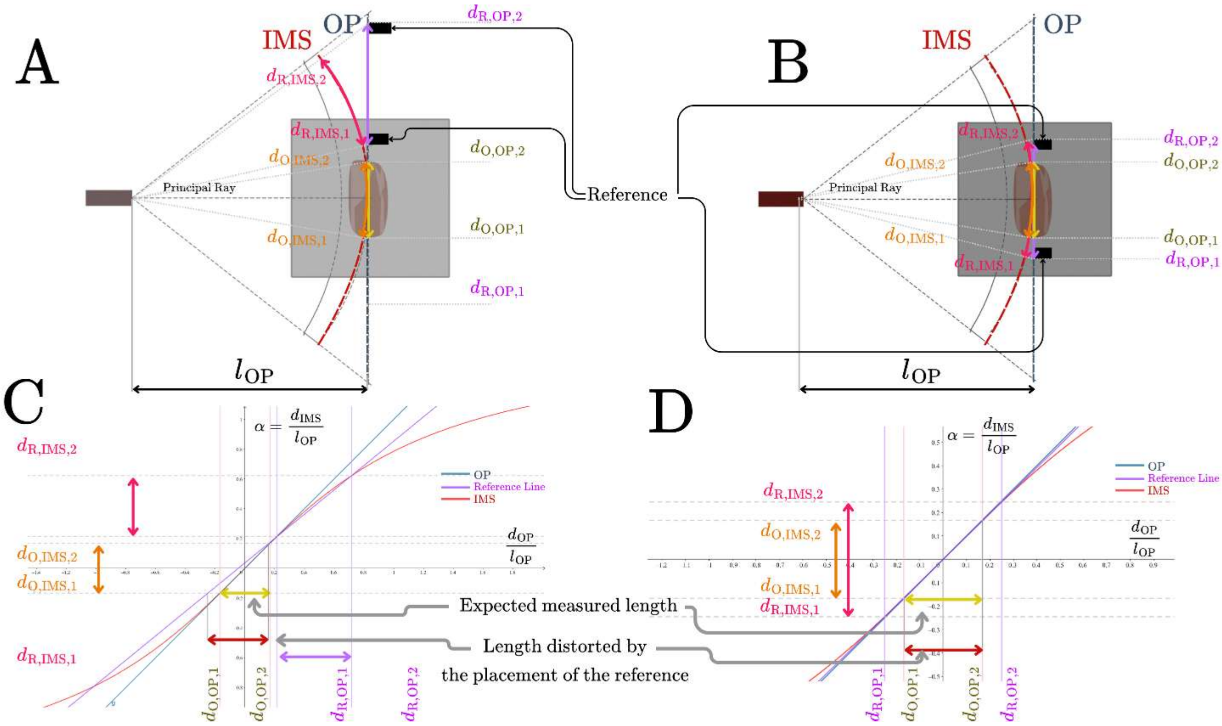


Figure 9: Depiction of different distortions depending on the placement relative to the image center (A – far from the center, B – symmetrically, C and D are their representations in view angle distortion graphs).

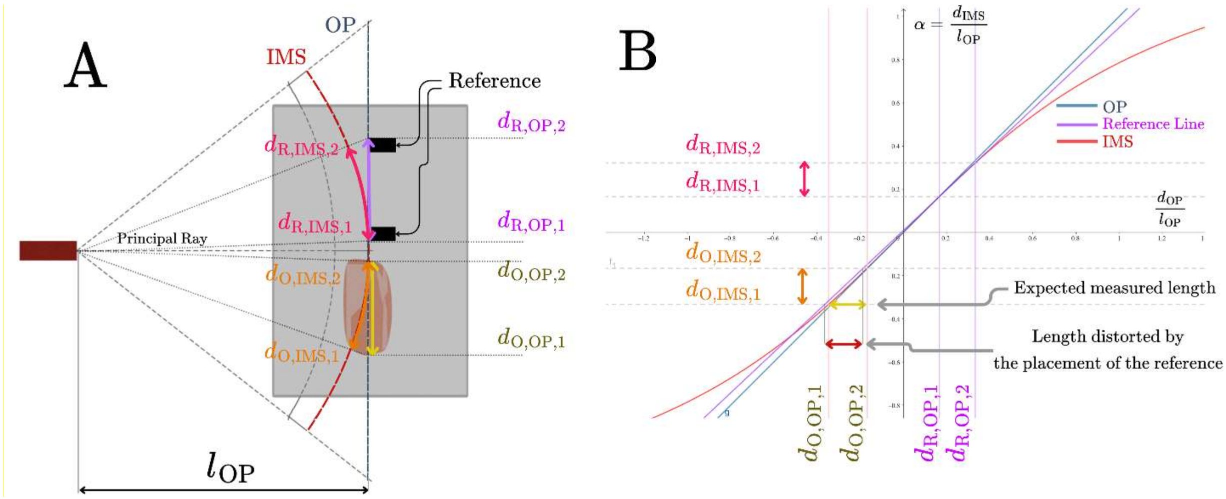


Figure 10: Ideal placement of a reference (A) in a 1D angle view distortion graph (B).

From these observations, five practical rules can be derived:

1. The camera must be placed sufficiently far from the measured object (distance can be estimated)
2. The camera must be placed in a way the object plane is known.
3. Reference must be placed within the object plane.
4. Reference must roughly be of the same length as the measured object.
5. If possible, place the reference and object so that in the image they are roughly the same distance from the image centre.

Even though all the calculations were made for 2D scenes, the general recommendations can be applied for 3D scenes as well. Another solution to address potential errors is to create an image filter which would distort the image depending on the known distance of OP from camera.

The last major systemic error comes from the fact that the shape of the object can obfuscate its real characteristic length. In figure 12, a circle radius is measured. The image projection, however, is a collection of points of intersections with camera rays. In this case, the edges of the circle in the image are represented by a collection of tangent points with camera rays. These points are sometimes situated closer to the camera than the measured radius. The measured length will in this case be longer. Therefore, it is important to get an indication of where the intersection of the object with OP is. In some cases, however, the indication is impossible. This error is partially mended by the distance of the camera from the object. Let's say the object has a radius $r_{\mathrm{O}} = 10$ cm. If the camera was $l_{\mathrm{OP}} = 60$ cm from its center, the projected length can be calculated by solving a set of equations derived from a right triangle:

$$l_{\mathrm{OP}}^2 = r_{\mathrm{O}}^2 + s_{\mathrm{l}}^2 \Rightarrow s_{\mathrm{l}}^2 = l_{\mathrm{OP}}^2 - r_{\mathrm{O}}^2 \tag{17}$$

$$r_{\mathrm{O}}^2 = s_{\mathrm{r}} \cdot s_{\mathrm{l}} \Rightarrow s_{\mathrm{r}} = \frac{r_{\mathrm{O}}^2}{s_{\mathrm{l}}} \Rightarrow s_{\mathrm{r}}^2 = \frac{r_{\mathrm{O}}^4}{s_{\mathrm{l}}^2} = \frac{r_{\mathrm{O}}^4}{l_{\mathrm{OP}}^2 - r_{\mathrm{O}}^2} \tag{18}$$

$$r'^2_{\mathrm{O,OP}} = r_{\mathrm{O}}^2 + s_{\mathrm{r}}^2 = r_{\mathrm{O}}^2 + \frac{r_{\mathrm{O}}^4}{l_{\mathrm{OP}}^2 - r_{\mathrm{O}}^2} = \frac{(l_{\mathrm{OP}}^2 - r_{\mathrm{O}}^2)\cdot r_{\mathrm{O}}^2 + r_{\mathrm{O}}^4}{l_{\mathrm{OP}}^2 - r_{\mathrm{O}}^2} = \frac{l_{\mathrm{OP}}^2 \cdot - r_{\mathrm{O}}^4 + r_{\mathrm{O}}^4}{l_{\mathrm{OP}}^2 - r_{\mathrm{O}}^2} = \frac{l_{\mathrm{OP}}^2}{l_{\mathrm{OP}}^2 - r_{\mathrm{O}}^2} \tag{19}$$

$$\delta_{OB} = \frac{\arctan\left(\frac{r'_{O,OP}}{l_{\mathrm{OP}}}\right)}{\arctan\left(\frac{r_{\mathrm{O}}}{l_{\mathrm{OP}}}\right)} - 1 \tag{20}$$

Using equations (19) and (20), the relative error due to the obstruction by the object δ_{OB} is for this case 1.34%, meaning a measured length would appear 1.34 % longer. Note that such calculation was possible only in this specific scene and in practice the error is hard to predict. To address this, it is important to know or indicate in the scene where an intersection of an object with OP is. In case of dough/bread, an apparatus can be made, where such intersection is indicated by strategically placed line lasers (see Figure 13).

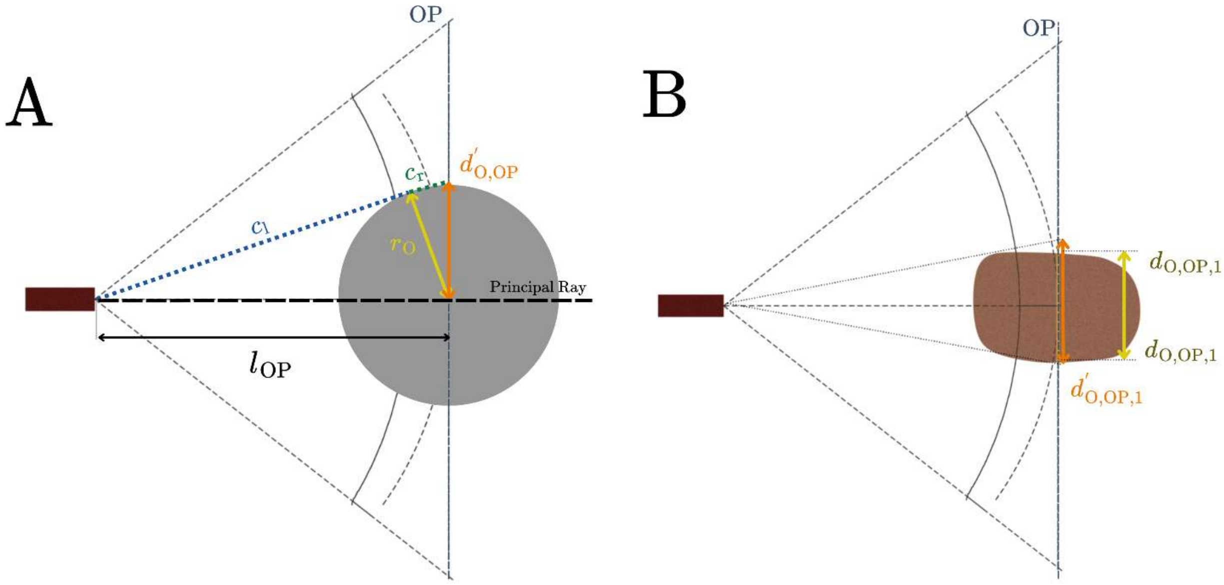


Figure 11: Depicted error in characteristic length measurement due to the obstruction by the object (A – circle, B – bread/dough).

The suggested design of an apparatus is depicted in figure 13. The apparatus is a solid rectangular frame with three laser pointers fixed onto the construction. For a XZ and YZ view (Figure 12G and Figure 12H), the frame itself serves as a reference and two lasers mark an upper intersection with OP. A bottom line is found via continuation of laser lines on the ground an object is placed on. For the XY view (Figure 12E and Figure 12F), one laser pointer determines an OP, and a different construct serves to place a reference in the OP. The camera distance is chosen in a way so that the laser light on the edges is visible, indicating that the projection of an object is close to the real intersection.

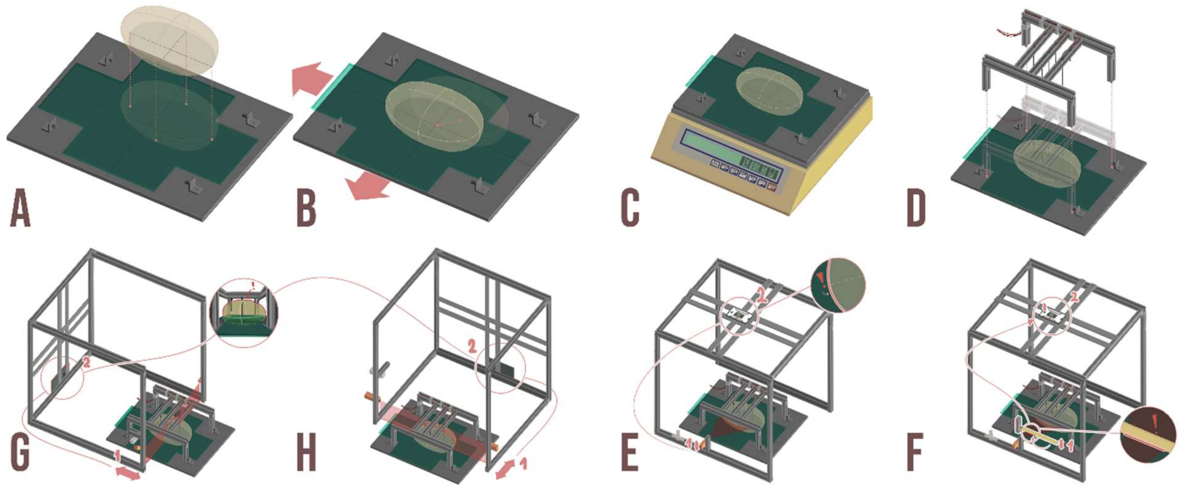


Figure 12:Methodology of data collection for image analysis via designed apparatus.

The apparatus was evaluated in a laboratory setting and in a bakery for sufficient speed as well. With this design a user was capable of measurement times below 2 minutes in this specific setting, sufficient for further implementation in the main data collection in bakeries.

Figure 13: Length measurement in-situ.

In conclusion, when determining dimensions and rough shape of an object via image analysis from limited number of images, A number of the systemic errors caused by the improper measurement setup and the obstruction from the shape itself were presented. From the observations derived above, an apparatus and general rules of setting of a scene were designed, evaluated, and improved upon to address those issues. In the setting of measuring dough/bread during a real process in the bakery a relative accuracy in single digits and sufficiently low times of measurements were achieved, resulting in output suitable for further processing. The dimensions and shapes of the objects (dough/loaves of bread) measured by this method can be used as an accurate validating data for mathematical models.

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